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13. ABSTRACT *(Maximum 200 words)*
This report describes development of a new hierarchical spectral basis appropriate for hp-finite element formulations on unstructured grids consisting of triangular and tetrahedral subdomains. The new multi-resolution basis has the following properties: 1) Jacobi polynomials of mixed weights, 2) Semi-orthogonality, leading to great sparsity, 3) Hierarchical structure, 4) Generalized tensor products, 5) Mixed order expansions, which provides great flexibility in adaptive discretizations, and 6) Gauss-Jacobi quadratures that minimize errors in complicated geometries. The accuracy of this method has been tested in two and three dimensions. Importantly, numerical results verified that the new hierarchical basis exhibits convergence even for highly distorted meshes.

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Parallel Multi-Scale Algorithms and Applications to Turbulence
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Objectives

• Advance the state-of-the-art in multi-resolution algorithms and parallel software.

• Develop high-order adaptive discretizations on unstructured meshes for the Navier-Stokes equations (mostly incompressible) at high Reynolds number.

• Apply algorithms to parallel simulations of complex phenomena in turbulence and other transport phenomena.

List of Accomplishments

• Development of a new hierarchical specrtal basis constructed appropriate for hp-finite element formulations on unstructured grids consisting of triangular and tetrahedral subdomains. This basis uses non-symmetric highorder Jacovi polynomials and is the building block for the accurate (spectral) solution of partial differential equations in complex geometry computational domains.

• In more detail, the new multi-resolution basis has the following properties: (1) Jacobi polynomials of mixed weights, (2) Semi-orthogonality, which leads to great sparsity, (3) Hierarchical structure, (4) Generalized tensor products, (5) Mixed order expansion which provides great flexibility in adaptive discretizations, and (6) automated Gaussian-Jacobi quadratures which minimize errors in complicated geometries. The scaling of the eigenspectra of the first and second derivatives has been studied in detail in order to determine the conditioning of this basis. We have shown that the scaling of these derivatives is identical to the scaling in standard tensorial spectral bases. In addition, due to a new type of tensor product we have developed, the computational cost in function evaluations, squares, derivatives, etc. is similar to the tensorial spectral methods used for structured discretizations.

• The accuracy of the new method has been tested both in two- and three-dimensions. For smooth solutions we obtain exponential convergence irrespective of the triangulization. Based on this new formulation a flow solver (NEKTAR) was developed in order to be used in turbulence simulations. There are two important new features of this code that make it superior to our previous spectral element flow solvers. First, it is fully adaptive as h- and p-type refinement can be readily applied using the hierarchical structure of the basis. For example, each edge or face or the interior in each tetrahedron could have different modes according to resolution demands. Second, it is more efficient as more effective preconditioners can be constructed using multi-resolution type algorithms.

• An important numerical result was obtained verifying that that high-order accuracy can be obtained on standard unstructured CFD meshes. Moreover, unlike low-order discretizations which may require Delaunay triangulizations for convergence, the new hierarchical basis can be used in conjunction with highly distorted meshes without accuracy degradation.

• Development of a new projection formulation of hyperbolic and parabolic problems using a discontinuous Galerkin formulation. This allows L^2 expansions to be used even for second-order elliptic problems, which, in turn, provides a great discretization flexibility. For example, appropriate trial bases can be used in different subdomains to deal with singularities, discontinuities and sharp gradients while optimizing their efficiency locally without global inter-dependence.

• Extensions to two-dimensional *hybrid* discretizations, where both quadrilateral spectral elements and triangular elements are connected together to provide greater flexibility in discretization.

• Extensions to three-dimensional *hybrid* discretizations, where polymorphic elements can be used, i.e. hexahedra connected to tetrahedra or pyramids or prisms.

• A detailed study of the spectral convective operator on unstructured and hybrid meshes, with bounds and semi-analytical expressions derived for the growth of the maximum eigenvalue as a function of the spectral order. This is important in choosing appropriate time stepping algorithms for convection-dominated phenomena.

• Development of a Navier-Stokes solver on hybrid meshes and validation verifying spectral convergence/accuracy for prototype solutions on complex geometries.

• Development of an Arbitrary Lagrangian Eulerian (ALE) formulation for spectral methods on unstructured and hybrid mehes.

• Development of a new computing approch in simulation, namely steering computing, where the developed hierarchical spectral methods are used on grids changing continuously in time by following an adaptive procedure. We have developed and performed such steering computational procedures for parallel computers in conjunction with interactive graphics in order to introduce a new mode of Direct Numerical Simulation, i.e. the *dynamic DNS*.

Personnel

• Faculty: G.E. Karniadakis, Professor of Applied Mathematics
• Post-Docs: T. Matushima, S.J. Sherwin (part-time) and A. Beskok (part-time)
• PhD Students: I. Lomtev, T.C. Warburton and C. Evangelinos.

Publications

1. T.C. Warburton, S.J. Sherwin and G.E. Karniadakis, "Spectral basis functions for 2D hybrid hp elements", SIAM J. Scientific Computing, submitted.

2. I. Lomtev, C.B. Quillen and G.E. Karniadakis, "Spectral/hp methods for viscous compressible flows on unstructured 2D meshes", J. Comp. Phys., in press.

3. A.K. Bangia, P.F. Batcho, I.G. Kevrekidis and G.E. Karniadakis, "Unsteady two-dimensional flows in complex geometries: Comparative bifurcation studies with global eigenfunction expansions", SIAM J. Sci. Comput., voll. 18 (3), p. 775, 1997.

4. I. Lomtev and G.E. Karniadakis, "A Discontinuous Galerkin Method for the Navier-Stokes equations", Int. J. Num. Meth. Fluids, submitted.

5. Ma Xia and G.E. Karniadakis, "The spectrum of the turbulent near-wake: A comparison of DNS and LES", 1st AFOSR Int. Conference on DNS/LES, Ruston, LA, 1997 (invited paper).

6. A. Beskok and G.E. Karniadakis, "Modeling separation in rarefied gas flows", AIAA 97-1783, 4th AIAA Shear Flow Conference, Snowmass Village, CO, 1997.

7. I. Lomtev and G.E. Karniadakis, "A discontinuous spectral/hp element Galerkin method for the Navier-Stokes equations on unstructured grids", Proc. IMACS WC'97, Berlin, Germany, 1997.

8. I. Lomtev and G.E. Karniadakis, "Simulations of viscous supersonic flows on unstructured h-p meshes", AIAA 97-0754, 35th Aerospace Sciences Meeting, Reno, 1997.

9. Karniadakis G.E. and Sherwin S.J, "Spectral/hp Element Methods for CFD", monograph, Oxford University Press, 1997.

10. Karniadakis G.E. and Henderson R.D., "Spectral Element Methods for Incompressible Flows", CRC Handbook of Fluid Dynamics, to appear, 1997.

11. Crawford, C.H, Evangelinos, C., Newman, D.J. and Karniadakis, G.E., "Prallel Computations of Turbulence in Complex Geometries", Computer & Fluids, vol. 25, p. 667, 1996.

12. Sherwin, S.J. and Karniadakis, G.E., "Tetrahedral hp Finite Elements: Algorithms and Flow Simulations", J. Comp. Phys., vol. 124, p. 14, 1996.

13. Crawford, C.H. and Karniadakis, G.E., "Reynolds Stress Analysis of EMHD-controlled Wall Turbulence, Part I: Streamwise Forcing", Phys. Fluids, vol. 9(3), 1997.

14. Evangelinos, C. and Karniadakis, G.E., "Parallel CFD Benchmarks on Cray Computers", Parallel Algorithms and Applications, vol. 9, pp. 273-298, 1996.

15. Sherwin, S.J. and Karniadakis, G.E., "Triangular and Tetrahedral Spectral Elements", Houston Journal of Mathematics, p. 509, 1995.

16. Sherwin, S.J. and Karniadakis, G.E., "A New Triangular and Tetrahedral Basis for High-Order Finite Elemenet Methods", Int. J. Num. Meth. Eng., vol. 38, p. 3775, 1995.

17. Warburton, T.C, Sherwin, S.J. and Karniadakis, G.E., "Hierarchical Refinement Using Spectral/hp Triangles and Prisms", Proc. 6ISCFD, Lake Tahoe, Nevada, September 4-8, 1995.

18. Sherwin, S.J. and Karniadakis, G.E., "Adaptive hp finite elements on unstructured meshes", IX Int. Conf. on Finite Elements in Fluids: New Trends and Applications, October 15-21, 1995, Venice, Italy.

Interactions/Transitions

The PI was invited to present the AFOSR-sponsored research during the duration of this project at:

• ICOSAHOM'95 • IBM • MIT • Cornell Theory Center • Boston University • Virginia Polytechic Institute • Hong Kong University • International Conference on Parallel Algorithms and Applications (Wuhan, China) • Institute for Scientific Computing, Chinese Academy of Sciences • Institute of Mechanics, Chinese Academy of Sciences • University of Tokyo • University of Notre Dame • University of California Santa Barbara • Wright Patterson Air Force Base • City College/Levich Institute • AFOSR Boiling AFB DC • Nuclear Regulatory Commission • National Institute of Standards and Technology • AIAA Conference New Orleans • University of Rhode Island • University of Maryland/CTC • University of Cincinnati • Worchester Polytechnic Institute • Penn State University • MHD Conference, Dresden, Germany

• First AFOSR Conference on DNS/LES • ASME micro-Therm Workshop • 10th Domain Decomposition Conference.

The code NEKTAR has been disributed to more than one dozen Universities and Laboratories. Some of them include Cornell University, Penn State University, University of Wisconsin, Imperial College, North Carolina University, Florida State University, OAK Ridge Labs, Nielsen, Inc., Boeing Inc. etc.